

# Micro Relativity

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Z. Naturforsch. **52 a**, 593–599 (1997); received June 10, 1997

A new synthesis based on microscopic classical thinking is attempted in the spirit of the molecular-dynamics-simulation (MDS) paradigm. Leibniz's idea that joint scale transformations cancel out is invoked. Boltzmann discovered that a time reversal in the whole universe is undetectable from the inside. As a corollary, objective micro time reversals occur in the interface between a subsystem and the rest of the universe, whenever the former undergoes a time reversal. This is shown to occur in a generic class of Hamiltonian systems. The "microinterface" arrived at generalizes the macro frame of relativity to the micro realm. Micro relativity comprises Bohr's idea of an observer-relative complementarity and Everett's idea of an observer-relative state. As in relativity proper, a multiplicity of worlds (cuts) exist. For the inhabitants of an artificial MDS universe, therefore a radically new option is available: world change technology.

## 1. Introduction

The "relativistic question" was first posed by Anaximander in the West and by the (unknown) inventor of the Yin-Yang symbol in the East: Is the world substance or difference? It is a lovingly made internal cut through the whole, the inventors both insisted. The modern theory of relativity is consistent with those ancient claims. Since relativity is an intrinsically macroscopic theory, however, the question naturally arises of whether a micro analog exists as well. The first microscopic relative-state formulation of physics we owe to Boltzmann.

Boltzmann saw that the principle of entropy increase has an "overriding power" in the sense that it remains true from the inside even if the whole universe runs the other way in time [1]. In other words, two universes which differ only in their directions of time are identical when viewed from the inside:

$$U^+ \underset{i.}{=} U^- \quad (1)$$

where  $U$  stands for "universe" and  $+$  and  $-$  indicate the direction of time, respectively. The symbol " $i.$ " under the equality sign denotes equivalence from the inside.

The Boltzmann equivalence, (1), constitutes a special case of "Boscovich covariance" [2]. The latter

claims that only the difference between the dynamics of an internal observer and the rest of the universe has any significance for the observer. The fact that joint scale transformations of observer and rest cancel out was already known to Leibniz [3]. Boscovich then included forces and temporal change. Finally, Boltzmann discovered that even the direction of time is subject to the difference principle.

## 2. Micro Time Reversals

The power of Boltzmann's result, (1), is even greater than he himself realized. Consider a Hamiltonian universe which consists of two partial systems, observer (O) and rest of the universe (R):

$$U = O + R. \quad (2)$$

Here the plus sign denotes Hamiltonian decomposition. From (1) it follows that

$$O^- + R^+ \underset{i.}{=} O^+ + R^-. \quad (3)$$

The new equation, (3), states that a time reversal in the observer is equivalent to a time reversal in the external world.

This is a rather unexpected result. It means that if "I" were an oscillator in my universe, the rest of the universe would regularly invert its time course relative to me, objectively. "Micro time reversals" thus can exist as an objective feature of the whole world valid in the interface.

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### 3. Micro Time Reversals are Robust

But I am not an oscillator. The time reversals result, found valid above for oscillators, may therefore not persist. To find out whether micro time reversals are indeed robust, it is necessary to look at some more general cases in due order.

First, we assume that the observer is made up, not from a single oscillator as above but from many oscillators. The simplest case is clearly that of many identical oscillators, all with the same phase. Then the micro time reversals persist trivially; for the only difference to the previous single oscillator case is that the energy which is seemingly expended during a time reversal increases in proportion to the number of oscillators involved.

The second-simplest case is one in which the many oscillators have an “infinitely narrow frequency distribution”. This case differs from the previous one in but one respect: the phases may now be unequal – for example, random. It follows that the energy seemingly expended per time reversal is reduced to that of a single oscillator again. Otherwise the previous result of global micro time reversals persists.

The same fact can be put more formally – by having a look at the “phase correlation function relative to the initial phases” (PCFRIP or PCF for short). This function still decreases down to “minus 100 percent” after a half period of the oscillators. We therefore have found an observable that remains unchanged in all three cases considered so far. From the point of view of the above PCF, nothing has changed in the two transitions. The same function can therefore be used to monitor the effects of the next transitions to be looked at.

Next comes the third-simplest multi-oscillator case. It is that of a very thin (“finitely narrow”) frequency distribution. This time, the PCF no longer decreases down to “minus 100 percent” after a mean half period, but only to “minus 99 percent” (say).

The new situation is nevertheless still equivalent to the previous one – if in that case a certain amount of “phase noise” (flipping oscillators at random) has been admitted into the picture. Note that the introduction of a certain amount of phase noise per unit time is indeed an admissible perturbation in the previous case.

As a consequence, we still have a full time reversal – only with a certain amount of “phase noise”

overlaid. The present case therefore is still “essentially unchanged” compared to the previous ones.

If one looks very closely, one at first sight can detect a qualitative difference between the present case and the previous one. Whereas in the present case the phases drift progressively apart since the frequencies are no longer exactly equal, they remained fixed in the previous case. While this is correct, it also is inconsequential due to the presence of a “resetting” effect which is the same in both cases. Immediately after a time reversal, the new phase distribution is “endorsed” in the sense that all previous phase shifts are forgotten. This is a consequence of the following fact. Assume that from the last time reversal on, no further phase shifts would occur. Then a perfectly equidistant sequence of time reversals would follow from that moment on – just as if the present situation were a new original initial distribution. Therefore, it is a new original initial distribution. In other words, the different time courses which exist between the two cases as far as the future (beyond the first time reversal) is concerned do not make a difference, as far as the lawful occurrence of time reversals is concerned, since the next time reversal always only depends on what happened since the last. Therefore, once more the same result as before applies: a time reversal after a half mean period, and then another, and so on.

Finally, we come to the least idealized case in the present series of many equal pendulum-like oscillators. The frequency distribution of the oscillators, with the same mean frequency as before, is no longer “very narrow” now but rather is just broad enough to erase the effect. This is the case of a so-called “critical” frequency distribution. Of course, many critical distributions which destroy the micro time reversals exist – starting from sufficiently broad unimodal (single-maximum) ones and proceeding, via the bimodal ones, to a variety of even more complicated cases. However, the case of greatest interest in the present context clearly is a particular unimodal distribution: the Gaussian or, more specifically, the “Maxwellian” frequency distribution.

Criticality is reached whenever the above phase correlation function (PCF) fails to produce a “dip below zero” at its first broad minimum occurring after roughly a mean half period. The transition between “dip” and “no dip” is analogous to the better-known transition between a “damped-periodic motion” and an “aperiodic creeping process” encountered in oscillator theory. The aperiodic creeping

process just fails to “overshoot”. Similarly in the present case.

Something more can be said. Shortly before the minimum disappears, the “apparent phase noise” (valid at the minimum) approaches 50 percent. This is the maximum tolerable value for phase noise; for only if more than 50 percent of the oscillators reach antiphase, compared to their initial phase as valid at the last time reversal, can one still speak of a “time reversal with noise added”. Once that threshold has been surpassed, the phase noise has obliterated the autonomous oscillation and it no longer makes sense to speak of time reversals. Thus the disappearance of the dip is indeed the crisis that marks the end of the occurrence of the (up to that point stable) phenomenon of “majority time reversal”.

Unexpectedly though, the “crisis” last described is apparently never consummated in reality, in case we have a Maxwellian frequency distribution – no matter how broad (“high-temperature”) the latter may become [4]. If this conjecture is correct, the phenomenon of micro time reversals can be said to be highly robust indeed.

#### 4. Extension to 1-D MDS Universes

A more realistic class of Hamiltonian universes than that based on frictionless oscillators like pendulums is obtained if freely moving particles are admitted into the picture. The simplest case here is a one-dimensional artificial universe that is made up from equal-mass, mutually repulsive billiards that are moreover subject to an initial Maxwellian velocity distribution.

In this case the above situation is retrieved because each particle lives in a “cell” of the same mean length. This fact is easily verified. First, the random velocities present at the outset never decay under the central collisions which necessarily take place between equal-mass particles in one dimension. Each velocity therefore forms a straight line in the combined  $x$ - $t$  (positions-versus-time) diagram. It is reflected at the boundary while retaining its absolute angle with the horizontal. This is because in collinear 1-D universes, all collisions are always “central” as mentioned. Second, the “straight lines” just described are not actually continuous but are only piecewise-straight. If one looks at them at a higher magnification, one sees that whenever two lines meet to cross, the crossing is actually avoided: The “letter X” is replaced by a “letter

Y” so to speak. Therefore, indeed “cells” are formed in which each particle moves forever in a zig-zag manner.

Hereby the whole sequence of the cells stays perfectly ordered. Nevertheless each cell’s position “meanders” irregularly back and forth in the one-dimensional space. This meandering is “slow”, however. During a short enough time interval of the order of a mean half period, the cells are effectively “pinned” in space. Viewed on a short enough time scale, the system is therefore indeed equivalent to a chain of nonlinear oscillators which obey a Maxwellian frequency distribution each, as far as the successive half periods of each are concerned. Thus, the present problem has effectively been reduced to that of the previous Section.

#### 5. Two Numerical Conjectures

The system just described is amenable to numerical investigation. An appropriate explicit Hamiltonian is available [5]. It describes a 1-D universe with a “bent”: the universe consists of two frictionless tubes that meet at an angle. Any two of the many loaf-shaped particles living in the tubes can only meet at the “junction”. There, they interact, not in a straight line (central collision) but rather in a slanted fashion, dependent on which particle enters the collisional area first. Since the energy exchange depends sensitively on the point of impact, Sinai-type chaos occurs in the hard-potential limit [5]. By implication, a Maxwellian velocity distribution forms automatically.

The following conjecture deserves to be checked numerically, in an explicit version of the above formal universe. Is there a characteristic time interval “ $T$ ”, not far from a mean half period, after which a maximum number of “more than 50 percent” of the particles have on average inverted their directions of motion compared to their initial directions? This question may be called the “majority-time-reversal conjecture”. It corresponds to the “negative dip conjecture” of the preceding Section; for the present “velocity-correlation function relative to the initial velocities” (VCFRIV) is closely related to the above phase correlation function (PCFRIP).

A second property worth to be investigated numerically is the fluctuation of the minimum just described. The fluctuation arises only if the system is kept sufficiently far from the thermodynamic limit – that is,

if the particle number simulated is not too large. Secondly, both the temporal distance of the next actual minimum and its height fluctuate as a function of the initial point chosen along the system's unique trajectory. The question worth to be checked numerically using an exactly time-invertible algorithm [6] is: Does there exist a self-consistent natural partitioning of the time axis into black and white (forward- and backward-running) time slices?

This would be the case if there allways existed a "best sequence of bidirectional minimums". The latter would define a unique – momentarily valid – "sliding time window" of black and white stripes. This second question may be called the "local-zebra conjecture".

## 6. Extension to Higher-Dimensional MDS Universes

At first sight, any generalization to higher dimensions appears out of the question. The reason for this bleak statement is simple: The particles can pass by each other in more than one dimension. Hence the occurrence of "cells" is no longer possible and hence the multiple-oscillator picture of an observing subsystem can no longer be upheld.

Unexpectedly, the multiple-oscillator picture actually survives under a specific condition – that the particles are mathematically equal like solitons. This assumption of "classical indistinguishability", as it may be called, is indeed very natural. Think of the majority belonging to the lightest particle class (having the mass of the electron, for example). The mathematical equality assumed implies the existence of a symmetry in phase space. The latter is a consequence of the unusual dynamical property of "trajectorial multi-uniqueness" [7] which applies in the presence of permutation symmetry. This multiplicity of trajectories,  $N!$  in all, reduces the phase space to a single, well-defined "cell" which possesses only  $1/N!$ -th the volume of the original phase space valid without the symmetry.

This Boltzmannian result in phase space has consequences in real space. Specifically, the formation of "classical Pauli cells" is implied in real space [8]. The underlying mechanism, in real space, is the occurrence at certain points in time of an (in general pair-wise) "exchange of identities at a distance" between particles. It takes place under certain well-defined conditions of mutual configuration. These counterintuitive events occur only because an

infinitely (transfinitely) sharp mathematical assumption – perfect identity – has been made at the outset. Specifically, the events in real space occur whenever one of the "walls" of the many identical cells in phase-space (which all reduce to one due to their being identical) is hit by the cell's internal trajectory (and another one enters the cell). The corresponding conditions in real space have so far been characterized in all mathematical detail only in some simple special cases [8].

The result – the formation of single-occupant classical Pauli cells in real space – unexpectedly restores applicability of the multi-oscillators paradigm. Hence the principle of "majority time reversal" of the 1-D case is re-encountered (provided the quantitative Maxwellian conjecture introduced above carries through). At the same time, a new principle – "majority rotation reversal" – comes into play since the cells are no longer one-dimensional now. Combined, the two principles generate "objective circus movements of the external world" in the interface by virtue of Boltzmann's difference principle. As a consequence, mere time reversals are again among the implications; in addition, we have spin-like effects.

## 7. Implications for the Inhabitants

A number of classical assumptions and their consequences have been introduced, some of them still conjectural. A highly nonlinear interface ("relative state") is created by Boltzmann's principle in a generic class of Hamiltonian universes.

The two major predictions arrived at above were: existence of "majority time reversals" in the observer and existence of "majority rotation reversals" in the observer. From these two observer-internal properties, four consequences follow for the external world by Boltzmann's principle. Three of them are "special" implications and one is a "general" one:

- 1) The time reversals in the observer imply the existence of "energetic perturbations" in the external world as it is objectively valid on the interface.
- 2) The time reversals in the observer imply the existence of "velocity perturbations" in the external world as it is objectively valid on the interface.
- 3) The rotation reversals in the observer imply the existence of "spin-like perturbations" in the external world as it is objectively valid on the interface.

More detailed implications can be derived from the preceding three (like: "action noise", "velocity limit",



“spin-pairing” [9]). The latter in turn give rise to even more detailed implications (like: nonlocality, length contraction, tachyonic mirror solution, violation of CP symmetry [9]). To look at any of these in more detail is, however, of little use as long as the “general” (fourth) implication has not been squarely faced:

4) All of the above nonlinear distortions of the external world as it is objectively valid on the interface are, (a) observer-specific and (b) moment-in-time-dependent (zebra-specific).

This “observer-centeredness” would be the main implication of micro relativity.

## 8. Discussion

The idea of microrelativity is much more familiar under the name “complementarity principle” [10]. Niels Bohr realized that an “internal observer” of a classical universe is automatically subject to certain observational limitations [11]. The formal content of the idea of complementarity – the alternative of either position or momentum being accessible but not both – was taken from the commutator relations of quantum mechanics as discovered by Heisenberg. However, the same facts seemed to reflect to Bohr a more general and classical “principle”.

Bohr generalized the principle beyond physics (discovering that “justice and love” are complementary in the process, to mention only one example), but did not arrive at further physical implications [11]. The reason may have to do with the fact that he held a second classical belief – that the universe is classical also on the macroscopic level of “records”. This belief was based on his insight that any communication about the results of a measurement has to be performed in a natural language [10]. This fact apparently prevented Bohr from seeing the possibility of an observer-specific macro world.

Microrelativity can – as proposed above – also be arrived at as an implication of a classical Boltzmann universe. Bohr’s idea of a microscopically exact complementarity principle is thereby revived. However, at the same time Bohr’s second classical view unexpectedly cannot be upheld.

Exactly thirty years after Bohr, Hugh Everett arrived at a microrelativistic physics. He called it “relative state formulation” [12]. It is a microscopically accurate observer-relative physics. The only major difference to the present Boltzmannian approach is that the exo level presupposed by Everett

was quantum rather than classical. Surprisingly, the power of the Yin-Yang principle (the vast difference between cut and whole) is virtually unchanged no matter whether the exo layer is assumed classical or quantum.

If one re-reads Everett’s paper today, one finds all the ingredients of a typical relativistic theory present in it – like the distinction between “absolute world” and “cut”. Even the little detail that the many different cut-specific worlds were not called “worlds” but “branches” by Everett attests to this parallelism. For they are all considered as outgrowths of the same single exo world. The connection to Einsteinian relativity is made in a footnote in which Everett states that the Einstein-Podolsky-Rosen correlations [13] pose no problem in his theory – a fact which later forced John Bell to pay very careful attention to Everett’s theory.

Bell first realized that the many worlds (branches) need not exist side by side – as Everett’s theory is ordinarily understood – but may just as well exist consecutively. Each is then valid over a very short time span only [14]. The formalism admits this new interpretation – but how can anyone arrive at such a strange picture? Bell showed that since the past and the future are world-specific, that is, change along as the world is changed, there is no way to find out whether one has just been transported from one world into another; for memory has been changed along [14].

Bell maintained a self-mocking attitude when presenting this new formal implication of quantum mechanics. The repeated usage of the word “solipsistic” by Bell attests to this lightness of spirit. This charming fact should not detract from the depth of his reasoning, however. The latter clearly projects back to the theory of “invisible change” once put forward by Heraclitus in antiquity. Heraclitus’ fragments can be read as a comment on the causal theory of reincarnation put forward by an Eastern contemporary of his. Although Bell did not allude to the ancient connotations of his modern formalism, it is hard to avoid being reminded of his friendship with the Dalai Lama when one reads his paper, [14]. Of course, there is no way to prove that discussions with a living Buddha played a catalytic role in the finding of Bell’s new interpretation of Everettian micro relativity. (Taking the courage to ask the Dalai Lama for his own recollections is an option in principle.)

The major prediction of micro relativity is the existence of an observer-centered consistent world

at every moment. This prediction is the same in both micro-relativistic theories, the Everettian and the Boltzmannian one. In the classical MDS version, the same result follows from the sliding “zebra window” – that every exo-objective event is re-perturbed at every moment in a new way.

The nonuniqueness of worlds predicted by micro relativity leads to an important question: Can the non-uniqueness be exposed? That is, is the theory empirically falsifiable as a theory of the real world? The answer appears to be yes. A case in point is the “relativistic Bell experiment” [15]. It demonstrates that relativity is “stronger” than quantum mechanics in fulfillment of Einstein’s [13] claim. Validity of the commutator relations of quantum mechanics indeed breaks down if the Bell correlations survive under a condition in which the two observers with their measuring devices both recede in opposite directions from the source of the correlated photons. For then each measurement is the first in its own frame – so that the other’s measurement is perturbed by it. This “completion” violates the commutator relations of quantum mechanics [15]. The only way to rescue the empirical validity of the latter is to acknowledge the existence of “more than one quantum world” [16]. This fact makes micro relativity falsifiable. Everett’s theory thereby ceases to be an “interpretation” and becomes a fullfledged theory [16]. A version of micro relativity (either quantum or classical) thus indeed needs to be adopted on empirical grounds – or so it appears at the time being. Even though the relativistic Bell experiment has not yet been performed, the consensus today is that taking this action will not be necessary since a positive outcome – survival of the Bell correlations and hence violation of the commutator relations in the standard non-Everettian formalism – is not in doubt [16].

Let us finally mention an applied argument in favor of microrelativity. Microrelativity predicts the possibility of a new type of technology – world-change technology. While all previous technologies aimed at changing something inside the world, here the whole world becomes a candidate for change. The first successful demonstration that more than one world exists (accomplishable with the relativistic Bell experiment,

for example) would at the same time amount to an empirical demonstration that the world can be changed.

At this point an historical conjecture suggests itself: it appears possible that Everett was already aware of the exceptional power of micro relativity. Right after the publication of his single paper [12] in an open Journal, Everett left academia to become a member of the Pentagon where he worked for the last 26 years of his life [17]. He died “from smoking” in 1983 (John Wheeler, personal communication 1993). It is tempting to think that he spent his time working there on the new type of technology made possible by microrelativity. Manipulation of the interface – once one knows it exists – is an irresistible challenge. If anyone ever believed in the existence of the micro interface, it certainly was Everett. Building the *world bomb* is as “technically sweet” as building the atom bomb was (to quote Oppenheimer’s famous regretful remark), but unlike what held true for the latter, the inventor of the former can apparently retain control to ensure a world-improving, peaceful use. (Once more, it will take an unusual amount of courage to ask Vice President Al Gore to kindly endorse a “request for unclassification” concerning Everett’s classified work done at the Pentagon provided any documents indeed exist.)

Does microrelativity exist? It currently is only a game to be carried out with computers for some time to come. Even basic simulations (like the “zebra test”) have yet to be performed. Like everything else which sprang forth from Leibniz’s fertile mind, micro relativity has the character of a fairy tale. It is “up to the kids” to decide whether the story is unbelievable enough to be real.

#### Acknowledgements

We thank Joachim Peinke, Jürgen Parisi, Achim Kittel, Ulrich Hoyer, Michael Böhm, John Wheeler, David Finkelstein, John Casti, Ichiro Tsuda, Siegfried Zielinski, Hans Primas, Koichiro Matsuno, Kuni Kaneko, Yukio Gunji, Nils Röller, Uwe Fischer, Falk Fischer, René Talbot, Mohamed El Naschie, Dietrich Hoffmann, and Peter Weibel for discussions. For J.O.R.

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